What is optimization?

All Optimization problems are made up of three basic ingredients:

An objective function which we want to minimize or maximize.

For instance, in a manufacturing process, we might want to maximize the profit or minimize the cost.

In fitting experimental data to a user-defined model, we might minimize the total deviation of observed data from predictions based on the model.

In designing an automobile panel, we might want to maximize the strength.

Note: The objective function should be quantitative/
A set of unknowns or variables which affect the value of the objective function.

In the manufacturing problem, the variables might include the amounts of different resources used or the time spent on each activity.

In fitting-the-data problem, the unknowns are the parameters that define the model.

In the panel design problem, the variables used define the shape and dimensions of the panel.
A set of constraints that allow the unknowns to take on certain values but exclude others.

For the manufacturing problem, it does not make sense to spend a negative amount of time on any activity, so we constrain all the "time" variables to be non-negative.

In the panel design problem, we would probably want to limit the weight of the product and to constrain its shape.
• The optimization problem is then:
  • Find values of the variables that minimize or maximize the objective function while satisfying the constraints.

• Are All these ingredients necessary?

• **Objective Function** :

• Almost all optimization problems have a single objective function. (When they don't they can often be reformulated so that they do!)
• The two interesting exceptions are
• **No objective function.**
• In some cases (for example, design of integrated circuit layouts), the goal is to find a set of variables that satisfies the constraints of the model.
• The user does not particularly want to optimize anything so there is no reason to define an objective function.
• This type of problems is usually called a feasibility problem.
• **Multiple objective functions.**
• Often, the user would actually like to optimize a number of different objectives at once.
• For instance, in the panel design problem, it would be nice to minimize weight and maximize strength simultaneously.
• Usually, the different objectives are not compatible; the variables that optimize one objective may be far from optimal for the others.
• In practice, problems with multiple objectives are reformulated as single-objective problems by either forming a weighted combination of the different objectives or else replacing some of the objectives by constraints
• These approaches and others are described as multi-objective optimization.
• **Variables**
• These are essential.
• If there are no variables, we cannot define the objective function and the problem constraints.
• **Constraints**
  • Constraints are not essential.
  • In fact, the field of unconstrained optimization is a large and important one for which a lot of algorithms and software are available.
  • It's been argued that almost all problems really do have constraints.
  • In practice constraints are encountered in day to day operation of the enterprise and these constraints will have to be necessarily considered for achieving maximum profits and productivity.
  • It is applicable for all enterprises in the economic scenario.
Energy efficiency optimization

**Macro level**:  
In big corporations / enterprises having a number of process / power plants (e.g: National thermal power plants), energy efficiency optimization of the total organization will bring down the operating cost, as this approach increases energy efficiency, reduces fuel consumption / cost and pollution levels.

**National level**:  
At national level, energy resource mix may be optimized to meet the energy demand at minimum cost. This concept can improve national productivity substantially.
Unit level Energy optimization

At unit level energy efficiency optimization, total unit may be divided into systems, subsystems and equipments.

Their energy consumption / generation data is collected and evaluated.

Taking the actual constraints imposed, an optimization model is developed with the objective of minimizing energy consumption and at the same time without loss of production.

In modern optimization models, Sulfurous emissions are also incorporated in the model to optimize the energy resource mix that will meet all the requisites.

Since this is dynamic, the evaluation must also be carried out more frequently.
Need for Energy Efficiency Optimization

High energy cost produced from primary and secondary sources, warrant maximization of energy efficiency in production as well as consumption.

In operation of boilers, heaters, pumps, compressors, turbines etc % Load on the equipment has an impact on the energy efficiency of the system.

A typical load vs efficiency is shown in the next slide.

In these equipments, efficiency increases with load, reaches a maximum and then starts dropping down.

When a number of such equipments is in operation, as in boilers, an optimum operation of boilers must be chosen to minimize energy consumption and operating cost.
How to achieve this?

This can be achieved by a systematic Energy Auditing of various sections at both micro level and macro level.

For more details, refer to technical audit info enclosed in the directory.

Certain examples are presented in this section for understanding the optimization methodology.
Single variable optimization model

This model is of the form

$$\text{eff} = a \times x^2 + b \times x + c$$

where $a$, $b$ and $c$ are constants and $x$ is the % load on the equipment in operation and $\text{eff}$ is the % efficiency of the equipment.

The model is developed using the plant data or test runs in which the load is varied keeping all the other parameters constant.

The observed efficiency is calculated and the data entered in the model.
## Example

<table>
<thead>
<tr>
<th>Load %</th>
<th>Equipment Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>60.0</td>
</tr>
<tr>
<td>70</td>
<td>70.0</td>
</tr>
<tr>
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<td>110</td>
<td>83.5</td>
</tr>
<tr>
<td>120</td>
<td>80.0</td>
</tr>
</tbody>
</table>
Model

Click on the arrow to run the model.
Type password as dr.ggr
The input file name is optmdl1
Don’t enter file extension name.
You may create a file of your own and run the program.

Click on the arrow to run the program.
Enter
Code: DR.GGR or dr.ggr to execute the program.
Observed Data vs Model

![Graph showing observed data vs model for efficiency vs load percentage. The graph plots efficiency (%) on the y-axis and load (%) on the x-axis. Two lines are shown: one for actual data (blue line) and one for the model (red line). The graph peaks around 80% load with an efficiency of approximately 85%.](image)

- Efficiency %
- Load %
Using models in optimization of total system

These simple models are very useful for energy efficiency optimization of the total system.

Consider 4 boilers B1, B2, B3 & B4

Load on boilers x1, x2, x3 & x4 % on design.

Eff % on each model is of the form $e_1 = a_1 x_1^2 + b_1 x_1 + c_1$

Similar models are generated for other boilers.

Fuel consumption for each boiler = $f_1 = \frac{(x_1/100) \times d_1 \times \text{enthalpy} \times (1/cv) \times (100/e_1)}{cv = \text{fuel calorific value in kcal/kg, enthalpy of steam = kcal/kg steam produced, } d_1 = \text{design capacity in kg/hr, } e_1 = \text{eff } \% \text{ from model}}$
Total optimization model

Minimize $f_1 + f_2 + f_3 + f_4$

subject to

$x_1 + x_2 + x_3 + x_4 = \text{demand}$

$x_1 \geq 0.6 \times d_1$

$x_2 \geq 1.0 \times d_2$

$x_2 \leq 1.105 \times d_2$

etc
Two variable model

In this case two opposing parameters have an impact on efficiency, energy loss or operating cost. The objective is to optimize the common parameter that will minimize the operating cost.

Typical example is the insulation thickness.

As the insulation thickness for the same service increases, the fixed cost component increases.

At the same time, energy loss reduces and cost of energy loss reduces.

Total cost = (cost of insulation + energy loss cost )\_\text{annualized}

The data can be converted into models and optimization carried out.
Insulation thickness optimization model

For developing this model, certain calculations will have to be made.

- Total insulation cost for various thickness of insulation.
- Annualization of the fixed cost taking into consideration the life of insulation and interest on capital.

This cost varies with

- Type of insulation material
- Life of insulation material
- Operation & maintenance costs
- Insulation efficiency
Cost of energy loss for various insulation thickness. This may be calculated by energy loss divided by cost of energy in US$ / Pounds etc per year.

The loss reduces with insulation thickness.

Cost of energy loss is dynamic and varies with the fuel cost. Hence it is imperative to carry out optimization of the insulation systems more frequently, especially when the energy cost goes up disproportionately.

Next slide shows the impact of insulation thickness on operating costs of the system.
Impact of insulation thickness on operating costs

- Plot note fixed cost of insulation increases with thickness and cost of energy loss reduces.
- Optimum thickness in this case is 40 mm at which the total cost is 110 thousand us$/yr
- This varies from system to system
Mathematical model

• This may be converted into a mathematical model by using regression methods.
• These models are
• Insulation cost model
• Energy loss cost model and
• Total cost model
• Data used in the program is given in the next slide.
## Data used in the model

<table>
<thead>
<tr>
<th>Insulation thickness mm</th>
<th>Cost of insulation</th>
<th>Cost of energy loss</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>120</td>
<td>130</td>
</tr>
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<tr>
<td>80</td>
<td>78</td>
<td>56</td>
<td>134</td>
</tr>
</tbody>
</table>
Methodology

• There are two methods by which the optimum insulation thickness may be arrived.

• In the first method, two models are developed for the insulation cost and energy loss cost. The first model is linear and the second model is non-linear.

• These two models are integrated to form the third and final model.

• This model is used for determining the optimum insulation thickness. ( use regression equations to develop the models )
Models

Insulation cost model:
This is of the form \( C_1 = A \times x + B \)

Cost of Energy loss:
This is of the form \( C_2 = A_2 \times x^2 + B_2 \times x + C_2 \)

Total Cost model:
This is equal to:
\[
C_1 + C_2 = C = A_2 \times x^2 + x \times (A + B_2) + (B + C_2)
\]
Where \( A, A_2, B_2, C \) & \( C_2 \) are constants and \( x \) is the insulation thickness in mm.
Models for insulation system

Insulation cost model

\[ C_1 = 0.9845237 \times x - 0.9285644 \text{ (S.E. 0.5989)} \]

Cost of Energy Loss model

\[ C_2 = 1.610116 \times 10^{-2} \times x^2 - 2.350891 \times x + 143.0446 \text{ (S.E. : 2.4601)} \]

Total cost model \( C_t = C_1 + C_2 \)

\[ = 1.610116 \times 10^{-2} \times x^2 - 1.366673 \times x + 142.1160356 \]

For determining minimum total cost, total cost function is differentiated WRT \( x \) and equated to 0.

When second order differential is –ve, then \( x \) represents the thickness at which the total cost is minimum.
\[
\frac{dc_t}{dx} = 2 \times (1.610116 \times 10^{-2}) \times x - 1.366673 = 0
\]
i.e. \( 2 \times 0.01610116 \times x = 1.366673 \)
i.e. \( x = \frac{1.366673}{2 \times 0.01610116} = 42.44 \text{ mm} \)

\[
\frac{d^2c_t}{dx^2} = 0.03220232 \quad (+ \text{ve})
\]

Hence \( c_t \) is minimum at \( x = 42.44 \text{ mm} \)

Total cost \( c_t = 1.610116 \times 10^{-2} \times (42.44)^2 - 1.366673 \times (42.44) + 142.1160356 \)

\[
= 113.115 \text{,000 US$}
\]

Refer to earlier slide and note the minimum point is between 40 to 45 mm thickness.
Time dependant model

• While the model given in this section refers to a totally new scheme, there is a need to change / replace insulation after certain period of time, because of the deterioration of insulation material and increase in heat loss.

• Next figure shows the optimum replacement time for the insulation, based on the energy loss data.
When the number of variables are more, the objective function will be generated using the operating data and converting them into appropriate models.

Then these models may be used in LP or NLP programs to optimize the objective function, within the stipulated constraints.

Refer to the book ‘Practical Energy Efficiency Optimization’ by Dr. G. G. Rajan for more details.